PHYSICS C

AP PHYSICS C

美国大学先修课物理学C

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AP Physics C: Mechanics Review Notes

Including AP Physics 1 and AP Physics C: Mechanics Knowledge Points

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Unit 1: Kinematics 运动学

1. 位置和位移 Position and displacement

a. 位置 Position

指物体某一时刻在空间的所在处。

b. 位移 Displacement（矢量 vector）

位置的变化量，是由初位置到末位置的有向线段。

2. 速度和速率 Velocity and speed

a. 速度 Velocity（矢量 vector）

-平均速度 average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

-瞬时速度 instantaneous velocity（位移对时间的求导）

$$v = \frac{dx}{dt}$$

b. 速率 Speed（标量 scalar）

-平均速率 average speed:

$$s_{avg} = \frac{\Delta d}{\Delta t}$$

-瞬时速率 instantaneous speed（瞬时速度的大小）

3. 加速度 Acceleration（矢量 vector）

a. 平均加速度 average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

b. 瞬时加速度 instantaneous acceleration

$$a = \frac{dv}{dt}$$

速度对时间的求导，位移对时间的二阶导

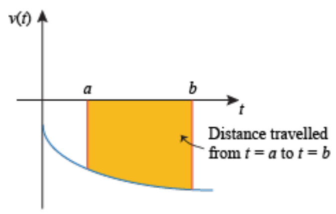
c. Four formulas if acceleration is constant

$$v = v_0 + at \quad \Delta x = v_0 t + \frac{1}{2}at^2 \quad 2a\Delta x = v_t^2 - v_0^2 \quad \Delta x = \frac{v_0 t + vt}{2}$$

4. 运动分析中的图形交互 Graphical interaction in motion analysis

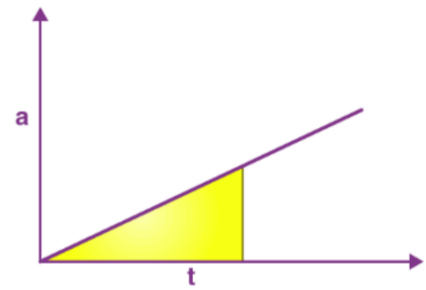
a. 积分速度 integrating velocity

$x_1 - x_0 = \int_{to}^{t1} vdt$ area between velocity curve and time axis from a to b



b. 积分加速度 integrating acceleration

$v_1 - v_0 = \int_{to}^{t1} adt$ area between acceleration curve and time axis from a to b



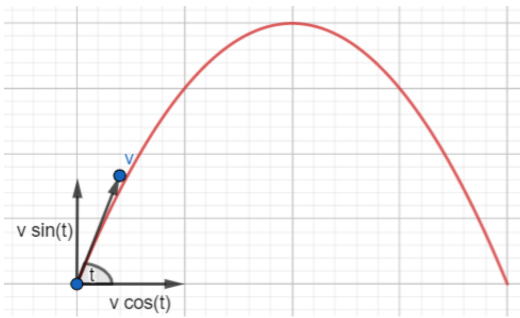
c. 位移-时间，速度-时间与加速度-时间图 x-t, v-t & a-t Graph

-XT：只有斜率有意义，为 V

-VT：斜率与面积都有意义，斜率为 a，面积为 Δx

-AT：只有面积有意义，为 Δv

5. 抛物运动 Projectile



-实质上是匀变速直线运动

-公式

Horizontal motion:	Vertical motion:
$\Delta x = v_{ox}t$	$\Delta y = v_{oy}t + \frac{1}{2}(-g)t^2$
$v_x = v_{ox}$ (constant!)	$v_y = v_{oy} + (-g)t$
$a_x = 0$	$a_y = -g$
	$v_y{}^2 = v_o{}^2{}_y + 2\,(-g)\Delta y$

6. 匀速圆周运动 Uniform circular motion

a. 匀速圆周运动的加速度 acceleration of uniform circular motion

-方向 direction: radially inward

-大小 magnitude

$$a = \frac{v^2}{r}$$

-周期 period

$$T = \frac{2\pi r}{v}$$

b. 匀速圆周运动计算公式

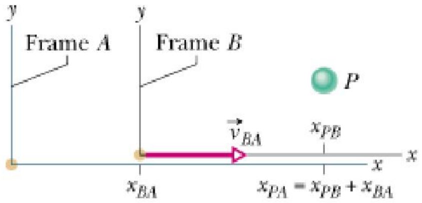
-线速度 linear velocity: $\frac{\Delta S}{\Delta t} = \frac{2\pi r}{T} = \omega r = 2\pi r n$

-角速度 angular velocity: $\frac{\Delta \theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$

-周期 period: $\frac{2\pi r}{v} = \frac{2\pi}{\omega}$

-向心力 centripetal force: $mr\omega^2 = \frac{mv^2}{r} = mr(2\pi)^2/T^2$

7. 一维上的相对运动 Relative motion in one dimension



The coordinate: $x_{pa} = x_{pb} + x_{ba}$
The velocity: $v_{pa} = v_{pb} + v_{ba}$
The acceleration: $a_{x,pa} = a_{x,pb} + a_{x,ba}$

Unit 2: Newton's Laws of Motion 牛顿运动定理

1. 牛顿第一定律 Newton's first law

a. 内容

任何物体都要保持匀速直线运动或静止状态，直到外力迫使它改变运动状态为止。

b. 惯性参考系 Inertial Reference Frames

-牛顿运动定律在其中能严格成立的参考系

-非惯性参考系 Non-inertial frame

例如：人在车上，以车为参考系，树在动（非匀直，静止），但树本身合外力等于 0

2. 牛顿第二定律 Newton's second law

a. 质量 Mass（标量 Scalar）

-物体所包含的物质的量的多少，是物体本身属性

b. 牛顿第二定律 Newton's second law

-内容

$$F_{net} = ma$$

-单方向上的牛顿第二定律 Newton's second law in one direction

$$F_{net,x} = ma_x$$
$$F_{net,y} = ma_y$$

-平动下的平衡状态 Equilibrium state under translation

$$F_{net} = 0$$

3. 牛顿第三定律 Newton's third law

作用力=反作用力

4. 一些特殊的力 Some particular force

a. 重力 gravitational force

-计算

$$F_g = mg$$

-重量 Weight

- 物体的重量是防止物体自由下落所需的净力的大小，由地面上的某人测量。
- W=mg
- 从地球到月球，mass 不变，weight 变

b. 支持力 normal force

-永远与接触面垂直

-由于支撑面发生形变，对被支持的物体产生的弹力

c. 摩擦力 friction

-静摩擦 static friction force

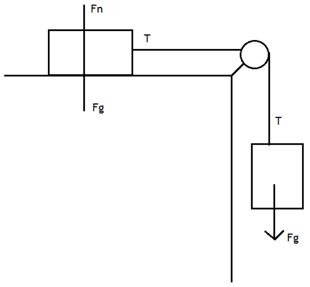
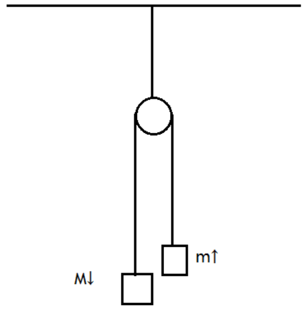
- 其他静摩擦力：利用受力分析与牛顿第二定律推算
- 最大静摩擦力：$f_{s,max} = \mu_s F_n$

-动摩擦 kinetic friction force

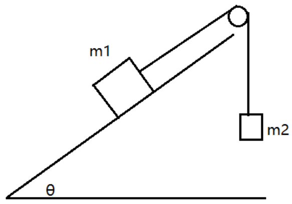
- $f_k = \mu_k F_n$
- 永远不变

5. 整体法解滑轮与斜面模型 Whole method (Pulley model and inclined plane model)

a. 滑轮模型 pulley model

条件：pulley, frictionless	
	
$a = \frac{m}{M+m}g$	$a = \frac{Mg - mg}{M+m}$
a=促进力-阻碍力/m 总	

b. 斜面模型 inclined plane model



$$a = \frac{m2g - m1gsin\theta}{m1 + m2}$$

6. 阻力与终极速度 The drag force and terminal speed

a. 阻力 drag force

$$D = \frac{1}{2}C\rho Av^2 = kv^2$$

v↑D↑

b. 终极速度 terminal speed

$$Vt = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2F_g}{k}}$$

c. How about V(t) when falls（有阻力 无摩擦）

-无动力 阻力与 V 成正比

$$ma = -kv \quad V = V_0e^{(-\frac{k}{mt})}$$

-无动力 阻力与 V^2成正比

$$ma = -kv^2 \quad \frac{1}{v} = \frac{1}{v_0} + \frac{kt}{m}$$

-有动力 阻力与 V 成正比

$$mg - bv = ma \quad v = \frac{(1 - e^{-bt})g}{b}$$

-有动力 阻力与 V^2成正比

Unit 3: Work, Energy, and Power 功，能量与功率

1. 动能 Kinetic energy

-定义式

$$K = 1/2mv^2$$

-单位: 1 joule=1J=1kg·m²/s²

2. 功与动能定理 Work and work-energy theorem

a. 功 work（标量 scalar）

-定义式

$$W = Fdcos\theta \quad W = \vec{F}\cdot\vec{d}$$

-work 可正可负（正功&负功）

-总功 Net work

方法一: 先求 net force，用公式 Fd 求得

方法二: 先求每个力的 work done，后加总得到

b. 动能定理 work-energy theorem

$$W_{net} = \Delta KE$$

3. 做功 Work done

a. 重力做功 Work done by the gravitational force

$$Wg = mgdcos\theta$$
$$Wg = -mgd(\cos 180°) = -mgd$$
$$Wg = mgd(\cos 0°) = +mgd$$

b. 弹力做功 Work done by the spring force

-弹力做功公式

$$Ws = \frac{1}{2}kxi^2 - \frac{1}{2}kxf^2$$

$$Ws = -\frac{1}{2}k\Delta x^2$$

-正功与负功

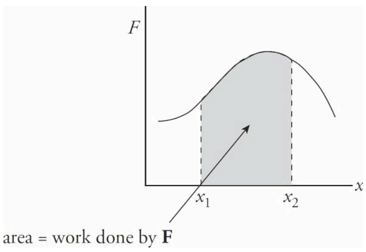
从离平衡位置远的地方到近的地方—正功

从离平衡位置近的地方到远的地方—负功

c. 变力做功 Work done by a general variable force

方法一：一维分析 one-dimensional analysis

F-x 图：面积为总功



方法二：动能定理计算

d. 外力做功 Work done by an external force

-外力做功=机械能变化量+摩擦力做功（生热）（耗散力）

-做了多少功就是转化了多少能量$W = \Delta Emec + \Delta Eth$

4. 功率 Power

-平均功率 average power：$Pavg = \frac{W}{\Delta t}$

-瞬时功率 instantaneous power：$P = \frac{dw}{dt} = Fvcos\theta$

5. 势能 Potential energy

-重力势能 gravitational potential energy

$$U = mgh$$

-弹性势能 elastic potential energy

$$U = \frac{1}{2}kx^2$$

- $U = -W$ 势能内部的力

6. 机械能守恒 Conservation of mechanical energy

a. 保守力与非保守力 Conservative force and non-conservative force

-保守力 conservative force

作用力所做的功不因为路径的不同而改变，则称此力为保守力（与初末位置无关的力），即重力与弹力。

-非保守力 non-conservative force

凡做功与路径有关的力称为非保守力。

b. 机械能守恒 conservation of mechanical energy

-条件：在一个封闭的系统里，只有保守力做功

-结论：$\Delta Emec = \Delta K + \Delta U = 0$；$KEf + PEf = KEi + PEi$

7. 势能曲线 Potential energy curve

a. 条件

在机械能守恒的条件下，一维下的运动

b. 力的分析 Force analytically

$$\Delta U(x) = -W = -F(x)\Delta x$$

$$F(x) = -\frac{\Delta U(x)}{\Delta x} = -\frac{dU(x)}{dx}$$

c. 拐点与均衡点 turning point and equilibrium point

-拐点 turning point

V 为 0，改变方向的点（U(x)图与机械能线的交点；机械能线之上的区域无法到达）

-均衡点 equilibrium point

V 不变后的一段（中性平衡 neutral equilibrium）

Unit 4: Systems of Particles and Linear Momentum 多物体系统与动量

1. 质心 Center of mass

a. 质心的位移 displacement of com

-由两个质点组成的系统

$$X_{com}=\frac{m1x1+m2x2}{m1+m2}$$

-由多个质点组成的系统

$$X_{com}=\frac{1}{M}\sum_{i=1}^{n}m1xi=\frac{1}{M}\int x\,dm=\frac{1}{M}\int_{a}^{b}x\frac{M}{L}dx$$

b. 质心的速度 velocity of com

$$P=MVcom$$

c. 质心的加速度 acceleration of com

$$Fnet=Macom$$

d. 对称性解质心 symmetry as a shortcut

利用图形分割和对称性解质心，若为点，在图形几何中心上；若为线，在图形对称轴上。

2. 线动量与冲量 Linear momentum and impulse

a. 线动量 linear momentum

-$P=mv$

-$Fnet=\frac{dP}{dt}$

b. 冲量 impulse

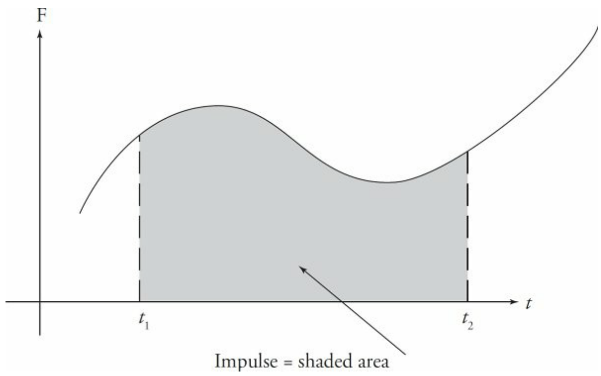
-力对时间的积分

$$J=\int_{ti}^{tf}F(t)dt$$

-$J=\varDelta P$

3. 力的积分 Integrating the force

-在 Ft 图中面积表示冲量，也就是动量的变化量



-$J=Favg\varDelta t$

4. 动量守恒 Conservation of linear momentum

a. 条件

$$Fnet = 0$$

b. 结论

$$Pi = Pf$$

c. 一分为二的动量守恒（爆炸模型 Explosion model&枪打子弹模型 Gun and bullet model）

-一物体分开成为两个物体，合外力等于 0，动量守恒

-$0 + 0 = m1v1 + m2v2$

d. 合二为一的动量守恒（碰撞模型 Collision model）

-弹性碰撞 elastic collision

- 动能守恒：$K_{1i} + K_{2i} = K_{1f} + K_{2f}$
- 动量守恒：$p_{1i} + p_{2i} = p_{1f} + p_{1f}$

-非弹性碰撞 inelastic collision

- 完全非弹性碰撞 completely/perfectly inelastic
  - stick together
  - $m1iv1i + 0 = (m1 + m2)vf$
- 普通非弹性碰撞 general inelastic

e. 单方向上的动量守恒（水滴滴车模型 Water drop car model）

水滴具有重力，系统合外力不为 0，水平方向上不受力，水平上动量守恒。

Unit 5: Rotation 转动

1. 转动运动学 Rotational kinematics

a. 角位移 angular displacement

$$\Delta\theta = \theta f - \theta i$$

b. 角速度 angular velocity

-平均角速度 average angular velocity

$$\Delta\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

-瞬时角速度 instantaneous angular velocity

$$\omega = d\theta/dt$$

c. 角加速度 angular acceleration

-平均角加速度 average angular acceleration

$$\Delta\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

-瞬时角加速度 instantaneous angular acceleration

$$\alpha = \frac{d\omega}{dt}$$

d. 平动和转动物理量的联系 Relating the linear and angular variables

-以弧度表示的角位移 angular displacement expressed in radians

$$\Delta\theta = \frac{\Delta s}{r}$$

-线速度与角速度 linear velocity and angular velocity（转动物理量×r=平动物理量）

$$\Delta s = r\Delta\theta \quad \Rightarrow \quad \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \quad \Rightarrow \quad \overline{v} = r\overline{\omega} \quad \boxed{v = r\omega}$$

-周期 period

$$T = \frac{2\pi r}{v} \quad T = \frac{2\pi}{\omega}$$

-加速度 acceleration

- 线加速度&切向加速度 linear acceleration & tangential acceleration

$$\frac{dv}{dt} = r\frac{d\omega}{dt} \quad \Rightarrow \quad a = r\alpha$$

- 法向加速度&径向加速度 normal acceleration & radial acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r$$

- 加速度 acceleration (real acceleration of object)

$$a = \sqrt{a_t{}^2 + a_r{}^2}$$

e. Four formulas if angular acceleration is constant

$$\omega = \omega_0 + \alpha t \quad \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad 2\alpha\Delta\theta = \omega_t^2 - \omega_0^2 \quad \Delta\theta = \frac{\omega_0 t + \omega t}{2}$$

2. 转动动力学 Rotational dynamics

a. 扭矩 Torque

-计算

$$\tau = (r)(Fsin\varphi) = rF_t \qquad \tau = (rsin\varphi)(F) = r_{\perp}F$$

$$\boxed{\tau = \mathbf{r} \times \mathbf{F}}$$

-方向

- 逆时针 counterclockwise 为正
- 顺时针 clockwise 为负

-净扭矩 net torque

$$\tau_{net} = sum\ of\ the\ individual\ torques = I\alpha$$

-牛顿第二定律的转动形式 Newton’s second law in angular form

$$\tau_{net} = I\alpha = \frac{dl}{dt}$$

b. 转动惯量 Rotational inertia

-定义：刚体绕轴转动时惯性（回转物体保持其匀速圆周运动或静止的特性）的量度。

-物理意义：质量的分散程度，表示惯性

-计算

$$I = \Sigma m_i r_i^2 = \int r^2 dm = \int_a^b x^2 (\frac{M}{L}) dx$$

- 旋转中心为 x 的零点
- 积分上限为最左边的 unit，积分下限为最右边的 unit

-平行轴定理 parallel-axis theorem

$$I = I_{com} + Mh^2$$

- 新的转动惯量=在质心处的转动惯量+总质量×移动距离的平方

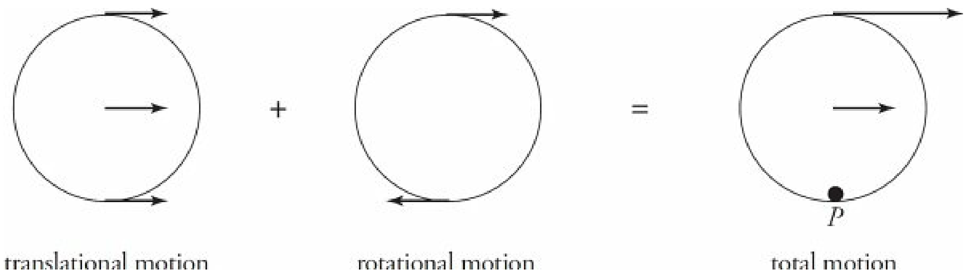
3. 旋转动能和与功的关系 Rotational kinetic energy and its relationship to work

a. 转动动能 Rotational kinetic energy

$$K = \frac{1}{2} I\omega^2$$

b. 滚动运动 Rolling motion

-平动和转动相结合的滚动 rolling as translation and rotation combined



- 顶部速度为质心速度二倍
- 底部无速度（静摩擦力）
- 打滑时，$a = r \times \alpha$不成立

-滚动的力与动能 forces and kinetic energy of rolling

$$K_{\text{rolling}} = K_{\text{translation}} + K_{\text{rotation}}$$

$$K_{\text{rolling}} = \frac{1}{2} m v_{cm}^{\ 2} + \frac{1}{2} I_{cm} \omega^2$$

-滚下坡道模型 rolling down ramp model

- 牛顿第二定理在平动上：$F = ma \;\rightarrow\; fs - Mgsin\theta = Ma_{com,x}$
- 牛顿第二定理在转动上：$\tau_{net} = I\alpha \;\rightarrow\; Rfs = I_{com}\alpha$
- $acom = \alpha R(rolling\ smoothly)$

总结公式

$$a_{com,x} = -\frac{gsin\theta}{1 + \frac{I_{com}}{MR^2}} \quad f_s = -I_{com}\frac{a_{com,x}}{R^2}$$

c. 功与功率 Work and power

-功 work

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

$$W = \tau(\theta f - \theta i)$$

-功率 power

$$Prot = \frac{dw}{dt} = \tau w$$

4. 角动量 Angular momentum

a. 一个质点上的角动量 Angular momentum for a particle

$$l = r \times P = r \cdot mv \cdot sin\theta$$

b. 一个刚体上的角动量 Angular momentum for a rigid body

$$L = Iw$$

c. 角动量守恒 Conservation of angular momentum

-条件

$$\tau_{net} = 0$$

-结论

$$Li = Lf \quad I_i\omega_i = I_f\omega_f$$

-角动量守恒的两种类型 Two types of conservation of angular momentum

- 不伴有碰撞的角动量守恒模型 Without collision

Model 1: 张开手臂模型 opening arms model

符合角动量守恒，$\tau_{net} = 0 \;\; I_i\omega_i = I_f\omega_f$

张开-质量分散-转动惯量↑

合上-质量集中-转动惯量↓

Model 2: 跳板潜水员模型 the springboard diver model

符合角动量守恒，$\tau_{net} = 0 \;\; I_i\omega_i = I_f\omega_f$

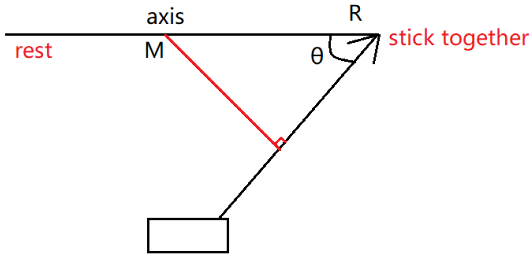
重力无 torque，因为其在质心上

Model 3: 跳远模型 long jump model

符合角动量守恒，$\tau_{net} = 0 \;\; I_i\omega_i = I_f\omega_f$

- 伴有碰撞的角动量守恒模型 With collision

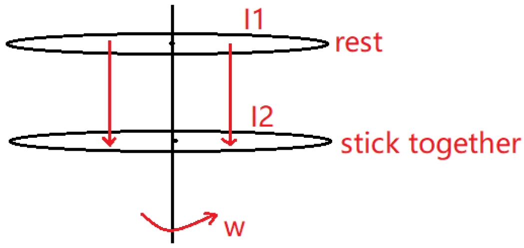
Model 4: 单摆撞杆模型 pendulum collision rod model



刚开始 rod 的角动量为 0；particle 角动量为$mvRsin\theta$

$$0 + mvRsin\theta = (I_{rod} + mR^2)\omega$$

Model 5: 两个圆盘相互碰撞模型 two discs colliding with each other model



$$0 + I_2\omega = (I_1 + I_2)\omega'$$

5. 平衡状态 Equilibrium

-平动平衡状态 translational equilibrium: $F_{net} = 0$

-转动平衡状态 rotational equilibrium: $\tau_{net} = 0$

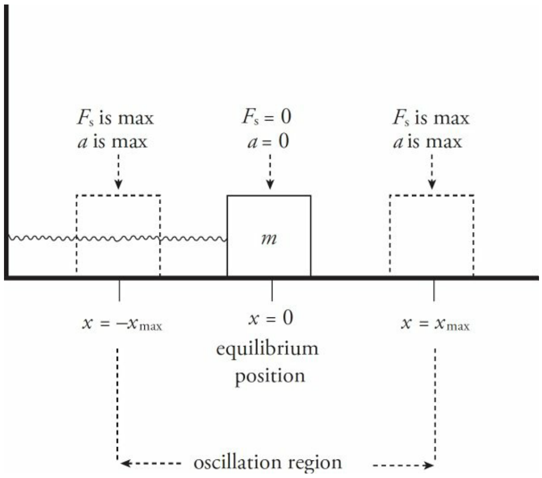
-总的平衡要求 the requirement of equilibrium: $F_{net} = 0$ $\tau_{net} = 0$

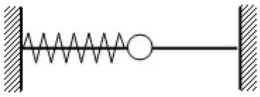
-静态平衡 static equilibrium

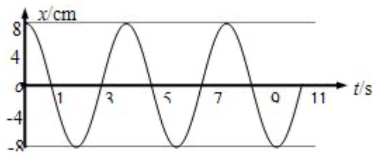
$\tau_{net} = 0$ does not mean that the angular velocity is zero; it only means that it's constant. If an object is at rest, then it is said to be in **static equilibrium**.

Unit 6: Oscillations 振荡

1. 简谐振动 Simple Harmonic Motion (SHM)







a. 定义

简谐运动是最基本也最简单的机械振动。当某物体进行简谐运动时，物体所受的力跟位移成正比，并且总是指向平衡位置。

b. 定义式

$$x(t) = Acos(\omega t + \varphi) \quad v(t) = \frac{dx}{dt} = -\omega Asin(\omega t + \varphi) \quad a(t) = \frac{dv}{dt} = -\omega^2 Acos(\omega t + \varphi)$$

A: 振幅 amplitude；ω: 角频率 angular frequency；φ: 位相常数 phase constant=0；$\omega t + \varphi$: 位相 phase

$$a = -\omega^2 A = -kx \quad \alpha = -\omega^2 \theta$$

2. 平动里的简谐振动 Simple Harmonic Motion in Translation

a. 简谐振动的速度和加速度 Velocity and acceleration of SHM

$$v_{max} = -A\omega \quad a_{max} = -A\omega^2 \quad \omega = \frac{2\pi}{T} = 2\pi f \quad T = \frac{1}{f}$$

b. 胡克定律与简谐振动 Hooke´s Law and SHM

-胡克定律与周期

$$F = -kx \quad k = m\omega^2 \quad \omega = \sqrt{\frac{k}{m}} \quad T = 2\pi\sqrt{\frac{m}{k}}(nothing\ with\ A)$$

-关于弹簧劲度系数 k

串联弹簧 In series: $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

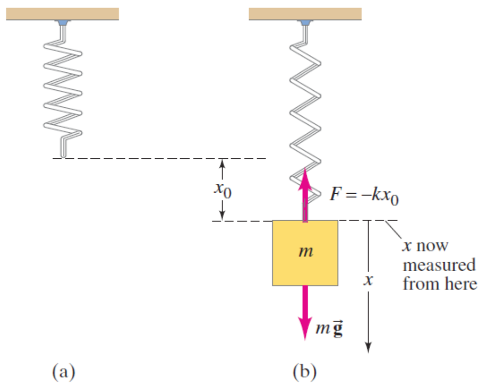
并联弹簧 In parallel: $k = k_1 + k_2$

c. 简谐振动的能量 Energy of SHM

-机械能计算：$E = U + K = \frac{1}{2}kA^2$

-机械能等于动能，势能为 0（最大速度与振幅的转换 Big formula）$E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

-竖直方向上的简谐振动 SHM in vertical direction



- The spring constant K is as same as in horizontal motion.
- The center of SHM is still at equilibrium which has changed.($\sum F = 0 = mg - kx_0$)
- No matter what the value of A is, whether it is horizontal or vertical, the period of SHM is always the same.$T = 2\pi\sqrt{\frac{k}{m}}$
- Big formula can also be used. $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

3. 转动里的简谐振动 Simple Harmonic Motion in Angular Form

a. 简谐振动的转动版本 Angular version of a simple harmonic oscillator

-胡克定律角动量形式 angular form of Hooke's law

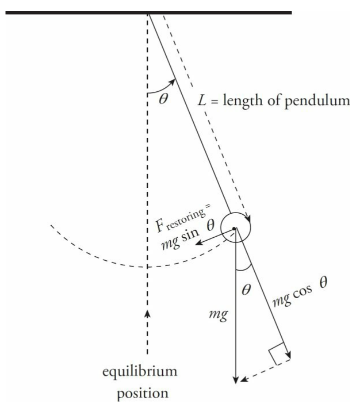
$$\tau = -\kappa\theta$$

-转动简谐振动周期 period of angular SHM

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

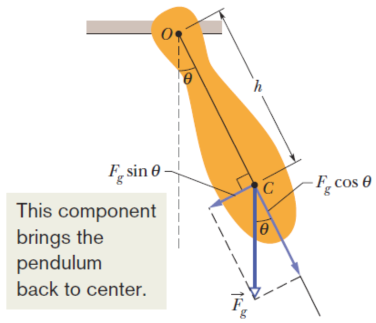
b. 应用：单摆与物理钟摆 simple pendulums and physical pendulums

-单摆 simple pendulums



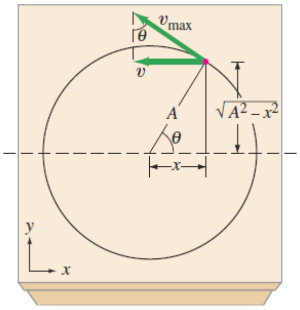
$$T = 2\pi\sqrt{\frac{L}{g}}$$

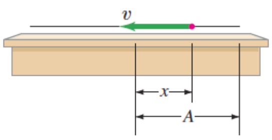
-物理钟摆 physical pendulums



$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

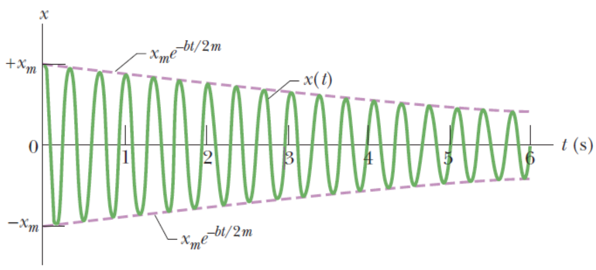
c. 简谐振动与匀速圆周运动 SHM and uniform circular motion





(b)

4. 阻尼简谐运动 Damped SHM



T=T1 周期不变

Unit 7: Gravitation 万有引力

1. 牛顿的万有引力定律 Newton's law of gravitation

a. 万有引力 gravitation

-大小

$$F = G\frac{m_1 m_2}{r^2} \quad G = 6.67 \times 10^{-11} m^3/kg \cdot s^{-1}$$

-方向

万有引力是物体见的相互吸引力，方向是质心连线方向。

$$F = G\frac{m_1 m_2}{r^2}\hat{r}$$

$\hat{r}$为半径单位向量 radius unit vector，决定万有引力方向。

b. 球壳理论 shell theorem

质量集中在物体质心处。

把微粒放在均匀的物质外壳 uniform shell of matter 中，合外万有引力等于 0。

c. 叠加原理 principle of superposition

$$\overrightarrow{F_{1,net}} = \overrightarrow{F_{1,2}} + \overrightarrow{F_{1,3}} + \cdots + \overrightarrow{F_{1,n}} = \sum_{i=2}^{n} \overrightarrow{F_{1,\iota}} = \int d\vec{F}$$

d. 内部万有引力 gravitation inside earth

$$F = G\frac{mM_{ins}}{r^2}$$

推导公式

密度公式：$\rho = \frac{M_{ins}}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3}$

内部万有引力公式：$F = G\frac{mM}{R^3} r = kr$

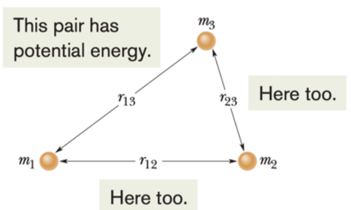
e. 万有引力势能 gravitational potential energy

-无限远星体之间的势能为 0

-两个星球之间的势能：

$$U = -\frac{GMm}{r}$$

-多个物体之间的万有引力势能



$$U = (-\frac{GM_1M_2}{r_{1,2}} + \frac{GM_1M_3}{r_{1,3}} + \frac{GM_2M_3}{r_{2,3}})$$

f. 由于大物体的重力加速度 acceleration of gravity due to large bodies

$$a_g = \frac{GM}{r^2}$$

2. 三大宇宙速度 cosmic speed limit

a. 第一宇宙速度（环绕速度）first cosmic velocity / orbital velocity

-航天器最小发射速度、航天器最大运行速度。

-大小为 7.9km/s

-证明方法为万有引力等于向心力。

$$\frac{GMm}{R^2} = \frac{mv}{R^2} \quad v = \sqrt{\frac{GM}{R}}$$

b. 第二宇宙速度（逃逸速度）second cosmic velocity / escape velocity

-人造天体无动力脱离地球引力束缚所需的最小速度。

-大小为 11.2km/s

-逃逸速度与机械能守恒

$$K + U = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0(\text{无限远处势能为 }0) + 0$$

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gr}$$

c. 第三宇宙速度（太阳的逃逸速度）third cosmic velocity / escape velocity

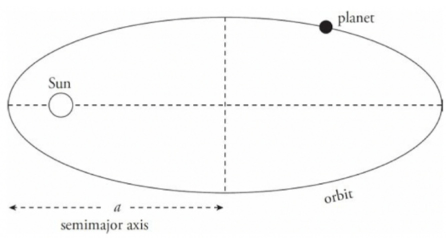
-指从地球起飞的航天器飞行速度达到 16.7 千米/秒时，无需后续加速就可以摆脱太阳引力的束缚，脱离太阳系进入更广袤的宇宙空间。这个从地球起飞脱离太阳系的最低飞行初速度就是第三宇宙速度。

-大小为 16.7km/s

3. 开普勒定律 Kepler's laws

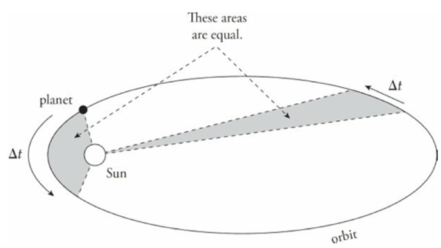
a. 开普勒第一定律（轨道定律）Kepler's first law / the laws of orbits

-所有行星绕太阳的轨道都是椭圆，太阳在椭圆的一个焦点上。



b. 开普勒第二定律（面积定律）Kepler's second law / the laws of area

-行星和太阳的连线在相等的时间间隔内扫过的面积相等。



-面积定律的两个守恒

- 机械能守恒 E=constant
- 角动量守恒 L=constant（合力矩等于 0，过旋转中心的力无力矩）

c. 开普勒第三定律（调和定律）Kepler's third law / the laws of period

-所有行星绕太阳一周的恒星时间的平方与它们轨道半长轴的立方成比例。

$$\frac{GMm}{r^2} = (m)(\omega^2 r) = \frac{2\pi}{T}$$

$$T^2 = (\frac{4\pi^2}{GM})r^3$$

4. 卫星轨道和能量 Satellites orbits and energy

-对于卫星来说，其机械能等于动能的相反数

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -K$$

-对于多个卫星轨道，如果其半长轴一样，则其机械能一样。